Project Mars: A Thorough Evaluation of a Manned Mission to Mars

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New Jersey Governor’s School of Engineering and Technology 2009

1 Abstract

For the past five decades, Americans and foreigners alike have obsessed over the wonders of space travel, racing to orbit Earth, land on the moon, and send satellites near and far to observe the mysteries of our universe. Recently, the exciting prospect of exploring Mars, our closest neighbor, arose. Satellite and rover missions to Mars have been so widely successful that the next logical progression is to send a manned mission to Mars. However, this task is difficult due to several factors, including the technological limitations, the small launch windows, and the risks that humans face when in space for extended periods of time. By using the 3D modeling software SolidWorks, however, we are able to model the components in our spacecraft and base as well as construct a real life model. Most importantly, we can calculate the mathematics behind the journey. With the proper resources, funding, and innovative technology, this endeavor could indeed be possible in the future.

2 Introduction

Over fifty years ago, in 1957, the Soviet Union launched the first artificial satellite into space. Just four years later, Russian Yuri Gagarin became the first human to enter space and orbit the Earth. A rapid “space race” between the United States and the USSR ensued, and in 1969, Neil Armstrong became the first man to set foot on the moon. Since then, however, there has been relatively little space exploration. NASA has made a number of trips to the moon, but no human has set foot on any other planet or extra-terrestrial object in our solar system. The Mars Exploration Rover mission sent two rovers, Spirit and Opportunity, to the Red Planet in 2003. Since then, the Phoenix lander has explored Mars as well. The missions were planned to last only 90 Martian days, or sols. However, they have lasted well over 2,000 sols, providing exciting and groundbreaking evidence for the many theories about Mars’s history and the possibility of extra-terrestrial life. With such successful missions, human exploration of Mars is the
next logical progression.

The valuable insight that can be gained from both exploration of the planet and research about its origins far outweighs the risks. As automated machines and technology become increasingly prevalent in our society, it has never been more evident that a machine cannot replace the reasoning skills of a human. Although the rovers have provided important data and discoveries about Mars, it is time that humans are given the chance to explore the Red Planet and provide scientists with even more information that can be used to explain some of the mysteries of our universe.

Our project is comprehensive. From

1. designing and modeling the base and spacecraft in SolidWorks to

2. calculating the trajectories required to reach Mars to

3. evaluating the risks involved in such a journey,

our work covers all the aspects in a manned mission to Mars.

3 Background

We use basic orbital mechanics to calculate and model the trajectories of our spacecraft on its way to Mars and back. Different velocities define unique orbits in space. Using this fundamental concept, we can model our journey from Earth to Mars.

There are two major types of orbits: closed and open. A closed orbit repeatedly retraces its path around a large body. The Earth, for example, revolves around the sun in a closed orbit. On the other hand, an open orbit will not be periodic and coasts to locations infinitely far from the larger body. Subsequently, there are two types of closed orbits: circular and elliptical. There are also two types of open orbits: parabolic and hyperbolic. Mathematically, these can be characterized by their eccentricities and energies. The eccentricity of an orbit is the curve’s deviation from a circle. Quantitatively, the eccentricity of a circle is 0; that of an ellipse is between 0 and 1; that of a parabola is 1; and that of a hyperbola is greater than 1. The energy of circular and elliptical orbits is less than 0; the energy of a parabola is 0; and the energy of a hyperbola is greater than 0.

Figure 1: Differences between four conic sections.
Elliptical orbits are found much more often than circular orbits. The reason lies in probability: it is much more likely for an orbit to have an eccentricity between 0 and 1 than an orbit to have an eccentricity of exactly 0. Since elliptical orbits are most prevalent, it is necessary to identify the major components of an ellipse.

The larger body is situated at one of two foci on the major axis. The closest point on the ellipse to the larger body is called the periapsis, while the farthest point is known as the apoapsis. Instead of using the gravitational constant $G$, we introduce a gravitational parameter equal to the product of the constant and the mass of a specific body and denote this by the Greek symbol $\mu$.

![Diagram of ellipse and its properties](image)

Figure 2: Diagram of ellipse and its properties.

When our shuttle delivers our spacecraft and crew to the International Space Station, the spacecraft will be assembled and put into a parking orbit. A parking orbit is one in which the satellite remains at a constant altitude over a body. This will increase our launch window, as the spacecraft can then revolve around the Earth more than once every twenty-four hours. When the spacecraft arrives on Mars, it will be put into an areosynchronous orbit above the planet. An areosynchronous orbit is one in which the period of the satellite and the planet Mars are the same; in addition, the satellite will remain above the same point on the planet and revolve around the planet at the same rate that the planet rotates.

We will also be using the Hohmann transfer, named after German physicist Wolfgang Hohmann, as a technique to help maximize the launch window as well as use as little engine power as possible. The reason for using the Hohmann transfer lies in its efficiency. While there are other techniques that take less time to send a spacecraft to Mars, the Hohmann Transfer is often the most popular because of its fuel efficiency. The technique involves a Hohmann transfer ellipse, which has the Earth at its perihelion and Mars at its aphelion. Two impulsive thrustings will be executed, first at the perihelion to stretch the orbit into an ellipse, and then a second firing at the aphelion to turn the elliptical orbit into a larger circular one, specifically, the orbit of Mars. Finally, the spacecraft will use a hyperbolic path to go into a parking orbit around Mars, from which it can enter the Martian atmosphere and finally land on its surface.

Furthermore, in order to enter this ellipse, we will use the Oberth maneuver with a hyperbola, named after Transylvanian physicist Hermann Oberth. With this maneuver, only one thrusting is needed to simultaneously escape the gravitational pull of the earth as well as leave a sufficiently high residual velocity at a point.
far away from the Earth in order to satisfy the velocity requirements for the Hohmann transfer ellipse.

4 Mission Timeline

1. Base parts are launched from Earth to the ISS
   - We will use a Delta II 7925H vehicle to launch our base parts into space.

2. Base is assembled at the ISS
   - The pieces of the base will be assembled and put on the payload of our unmanned rocket.

3. Base is launched for its trip to Mars
   - Spacecraft parts are launched from Earth to the ISS
   - As soon as the base is launched, the spacecraft that will send the crew.

4. Base lands on Mars and gradually expands through energy from solar panels
   - Solar panels absorb energy; when enough energy is absorbed, base begins to slowly assemble.
   - Spacecraft is assembled at the ISS
     - Once assembled, the base will collect and store solar energy.
   - Astronauts train on Earth
     - Astronauts must be in prime physical shape, be extremely healthy, and have an acute knowledge and understanding of the science they will be using when conducting experiments on Mars.

5. Astronauts take a rocket to the ISS
   - We will use a Delta II 7925H vehicle to launch our crew into space.

6. Astronauts launch with the spacecraft to Mars
   - The spacecraft that was assembled at the ISS after the base was deployed carries our crew and fuel safely to Mars. It takes about 259 days to reach Mars.

7. Spacecraft arrives in areosynchronous orbit around Mars
   - The spacecraft orbits Mars directly above the base.

8. Astronauts descend to Mars in a module
   - The module separates itself from the spacecraft and sends the crew to the surface of Mars near the base with a soft impact through reverse thrust, a parachute, and air bags surrounding the entire module to cushion the impact.

9. Astronauts live on Mars, conducting experiments
   - Astronauts stay on Mars for approximately 261 days.
10. Astronauts take module back to the spacecraft
   • The module will be equipped with enough fuel and power to get through the (relatively thin) Martian atmosphere and stop at the orbiting spacecraft.

11. Astronauts launch with the spacecraft to the ISS
   • The spacecraft will take approximately 259 days to return to Mars.

12. Astronauts take the rocket from the ISS back home to Earth
   • A Delta II 7925H vehicle’s module will bring the astronauts to Earth, landing in the Pacific Ocean.

5 Design Decisions

5.1 Base

Our base was designed to house our current crew and remain on Mars for missions to come. We designed the Martian base specifically to fit the needs of our crew and provide a safe and stable living and working environment. Furthermore, the division of rooms within cylindrical shapes was designed in order to maintain a compact design which could easily travel to Mars and to provide for easy access between the rooms. The base is three stories tall. On the first story, there is a research room and living quarters. The medical room, fitness center, fuel production center and oxygen/food/water storage center are all on the second story. The observatory/control tower is on the third story. Greenhouses surround the first and second stories in a dome-like shape around the base, and a waste storage cylinder occupies these two stories at the center of the base.

In the research room there will be a “garage door” through which the rovers can leave and enter, delivering data and samples of rocks and soil. The crew will be able to utilize important scientific equipment such as spectrometers, x-ray diffractors, and high-powered microscopes. The living quarters will contain beds, storage, showers, and more. The medical room will be equipped with all necessary basic equipment to ensure the health and safety of our crew in the case of disease or sickness due to the alien environment. Since keeping the crew in top physical shape is imperative to their survival on Mars, the fitness center will be equipped with a treadmill, free weights, exercise bikes, and room to stretch and jog. The observatory will contain a high-power telescope along with other instruments to study astronomy.
from Mars. The greenhouses surround the first and second stories in order to provide oxygen to most rooms so that in case of an emergency, the astronauts need not be forced into another room for oxygen. We feel that the components of our base are unique in that they will provide the first functional living and working environment away from Earth.

In order to keep the base compact for its travels through space and landing on Mars, the observatory/control tower is not initially located on top of the other rooms. At the International Space Station, it is constructed around the center of the waste storage cylinder and programmed so that on Mars – when the solar panels retrieve energy from the sun – the room will be powered to lift into its own third story. This also pushes the solar panels to a slant to maximize energy retrieval efficiency. As the observatory/control tower lifts, a collection of ladders and walkways, connected to the bottom of the room, will expand to allow access to all three floors. This “staircase,” enclosed in its own cylindrical wall, will also act as a hub between rooms on the same floor to ensure means of entry to any room despite contamination in any other room.

5.2 Spacecraft

![Side view of spaceship to Mars.](image)

Our spacecraft was designed with the goal of safely and comfortably transporting our astronauts between Earth and Mars. It is divided into three major sections, each increasing in size: the control room/escape module, the centrifugal living area, and the storage compartment.

5.2.1 Control room

The control room/escape module is located in the front of the ship, with the control room section inside the main spacecraft and the escape module cylinder protruding out of it. During the journeys between Earth and Mars, the control room and escape module will be open to one another, as the escape module will be used in accordance with the control room to operate the functions of the ship. At this point, the escape module will simply be used as an addition to the control room to allow
the astronauts to operate the spacecraft. Once the spacecraft arrives in orbit around Mars, the control room and escape module will be closed off from one another, allowing the escape module to break off. The astronauts will use this to land on the Mars surface, with the module containing all necessary controls and equipment for a safe descent. Remaining on the spacecraft will be the control room, which contains communication devices that allow the astronauts to contact it from the Martian base’s control tower. These messages can then be relayed from the spacecraft to the space agencies on Earth.

5.2.2 Living area

Connected to the control room/escape module is a larger cylinder, which serves as the main living/recreation area for the astronauts. Included in this cylinder are the living quarters, a medical room, a greenhouse, and an exercise room. The entire cylinder will act as a large centrifuge, rotating along a fixed axis which goes through the center of the cylinder between its flat faces. As the cylinder spins, centripetal acceleration is generated pointing to the center. The centripetal force will be realized in the normal force from the walls on the astronauts, simulating a feeling of gravity in the room. The round walls inside the cylinder will then act as the floor. This technique will help to prevent muscle atrophy, bone decalcification, shift of bodily fluids from the lower body to the head, and other negative side effects of an extended period in zero gravity. This rotation will also be programmable along the journey, and can be gradually slowed in order to shy away from Earth’s gravity and begin simulating Mars’s gravity. This will prepare the astronauts’s bodies for the change they will experience upon arrival at Mars.[14]

5.2.3 Storage

The storage compartment at the rear of the spacecraft is simply a large room. It will store the oxygen, food, water, and fuel that the astronauts will need to complete the journey. In addition to the provided food, the greenhouse inside the spacecraft will also be actively growing food because it is impractical to carry more than a year’s worth of food in the spacecraft. This area will also hold the waste that will be collected along the trip.

The escape module will be multifunctional. Besides using it to land on Mars, the astronauts will also use it to exit Mars’s atmosphere at the end of their time on Mars. The module will rejoin the spacecraft, in orbit around Mars, which will transport the astronauts on their voyage back to Earth.

5.3 Solar Panels

The Mars base design includes solar panels that will be a key source of power on the base. In Figure 3, they are shown as the silver panels on the slanted surface joining the two cylinders. The first objective of the
panels is to power the base for the observatory’s expansion into its own third story. As the base lands, the solar panels will immediately begin storing energy to use for the expansion. The solar panels will later power the conversion of Mars’s high concentration of carbon dioxide into carbon monoxide (for fuel) and oxygen. Any excess energy will be stored for future use.

“Bifacial” oblique solar arrays on Mars on the area surrounding the base are the most efficient design in that they are less massive yet generate more power than flat panels. We also have found through data from a NASA study that they will be efficient in converting enough solar energy into usable power for a manned Mars mission. Solar panels are also more reliable and less volatile than nuclear power. A common risk in using solar panels on a planet like Mars, however, is that the panels can easily be covered by dust due to Mars’s frequent dust storms. Since this mission is manned, the crew will be able to wipe off any excess dust to be sure that the panels are used to their optimal ability. Solar panels also do not affect the environment of Mars in any known way.\[15\]

5.4 Fuel Manufacturing

An important aspect of our mission, which requires an awareness of Mars’s environment, is fuel production. Mars contains relatively high amounts of carbon dioxide, so a device will be used to separate this \( \text{CO}_2 \) into \( \text{O} \) and \( \text{CO} \). This process will produce oxygen and carbon monoxide for use as a synthetic liquid fuel. The device will likely convert the \( \text{CO} \) into methanol for simple use on the base, as this is the easiest fuel to synthesize. However, it can also produce gasoline, diesel, or jet fuel. As soon as our base arrives on Mars’s surface and completes its expansion, it will begin converting these compounds by utilizing the solar panels to operate a reactor. The fuel that is produced will be used for the rovers that will be brought to Mars, for the module that will ascend to the spacecraft in Mars’s orbit, and for the spacecraft’s return to the ISS.

5.5 Landing Site

We have decided to land on Mars’s northern polar region. We chose this site because it contains larger amounts of carbon dioxide relative to the rest of the planet. We will be using this gas for fuel and oxygen manufacturing. Also, the site is one of the few areas on Mars believed to have once contained water.

6 Launch Calculations
Earth and Mars are collinear with the Sun approximately every 2.135 years. Thus, roughly every 2.135 years, there is a launch window. The last Mars Exploration Rover mission conducted by NASA launched rover Spirit on June 10, 2003, which is equal to approximately 2003.528 years. Thus, for integers \( n \), the times of the launch windows can be expressed in the form \( 2003.528 + 2.135n \). For our mission, the launch windows will occur at January 2029, February 2031 and March 2033 (\( n = 12, 13, \) and 14, respectively). The base, including supplies and life necessities, will be sent to Mars in 2029 to await the astronauts’s arrival in February of 2031. The preliminary launch will also serve as a test flight, so that adjustments can be made if it fails. Each journey will take approximately 8.5 months. We will be launching the spacecraft from the John F. Kennedy Space Center in Florida.

Initially, the most obvious flight trajectory would ideally follow a straight line from Earth to Mars when they are collinear with the Sun. However, this is impractical because of the massive amounts of fuel that would be required to directly fight against the gravitational pull from the Sun. More importantly, the trajectory would be bent due to gravitational effects and the result would be the spacecraft entirely missing Mars.

Therefore, to send our astronauts to Mars, we will use a hyperbolic escape path to put them into a parking orbit around the Earth, after which a Hohmann transfer will be used to send the spacecraft into a parking orbit around Mars.

First, we will send the satellite into a roughly circular orbit about the Earth at a low-Earth orbit altitude of \( h = 3.571 \times 10^5 \) m, in order to reduce mechanical and fuel requirements. This height is significant because it represents the altitude of the International Space Station, where we will be docking and assembling our spacecraft to take advantage of the low gravitational obstacles. After finishing the assembly at this altitude, we must launch the rocket with a velocity of 7738 m/s for it to enter the ISS orbit.

This launch velocity will provide a sufficient amount of centripetal force to keep the satellite at its orbit, so that the tangential velocity will be \( v_T = 7693 \) m/s.

![Diagram of Earth and Mars orbiting the Sun, orbits approximated as circles to simplify calculations.](image)

**Figure 5:** Diagram of Earth and Mars orbiting the Sun. Orbits are approximated as circles to simplify calculations.

![Diagram of Hohmann transfer used to send spacecraft from orbit of Earth to orbit of Mars.](image)

**Figure 6:** Diagram of Hohmann transfer used to send spacecraft from orbit of Earth to orbit of Mars.
orbits. The velocities of the spacecraft at the perihelion and the aphelion are 32724 m/s and 21472 m/s respectively.

To escape the Earth and enter the transfer ellipse from the perihelion, we can use the Oberth hyperbola to save fuel and energy. With a single firing of 3607 m/s at an appropriate angle, we will be left with a residual velocity of 2605 m/s at a point far away from the Earth in order to enter the ellipse.

By Kepler’s third law of planetary motion, the journey from the perihelion to the aphelion of the transfer ellipse takes approximately 8.5 months, meaning we must launch our spacecraft 8.5 months ahead of the launch window, when Mars is not collinear with Earth and the Sun yet.

Figure 7: Diagram of Oberth hyperbola used to escape from Earth.

Figure 8: Diagram of Oberth hyperbolic flyby used to enter an areostationary orbit about Mars.

After the ellipse, we must speed up with a hyperbolic flyby in order to enter the orbit of Mars. Since Mars revolves at a velocity of 24077 m/s, the spacecraft is moving 2605 m/s slower than Mars.

We also want the spacecraft to stay in an areosynchronous orbit above Mars to allow for easy communication between the craft and the base. Using Kepler’s third law of planetary motion, we find that the sufficient altitude is $2.0438 \times 10^7$ m.

In order to break from the transfer ellipse and enter Mars’s orbit simultaneously, we find that the spacecraft must travel 3315 m/s slower. However, since there is already a sufficient centripetal force to support a tangential velocity of 1450 m/s, the spacecraft only needs to decrease its speed by $3315 \text{ m/s} - 1450 \text{ m/s} = 1865 \text{ m/s}$. 
Finally, the total velocity budget for the trip is

\[8112 \text{ m/s} + 3607 \text{ m/s} + 1865 \text{ m/s} = 13584 \text{ m/s} \]

The total velocity budget is significant because acceleration is required to initiate these velocity changes. The acceleration roughly corresponds to the amount of fuel that will need to be used to fuel the spacecraft.

The impact speed will be approximately \( v_{\text{impact}} = 4850 \text{ m/s} \). To dramatically decrease this velocity, we will fire engines in the reverse direction and then use a grape-like landing apparatus similar to the one used for the Spirit and Opportunity mission to land at a low terminal velocity.

(A list of constants and formulas can be found in Appendix A, and more detailed calculations can be found in Appendix B.)

7 Mission Analysis and Risks

7.1 Health Risks

Health problems are some of the most significant risks in our mission. Recent work determined that humans could indeed survive in such extreme conditions. Four Russians, one Frenchman, and one German simulated a 105-day trip to Space where they were monitored and tested constantly to understand and measure their physical and biological reactions to the conditions.

Another experiment is planned to begin next year and last for 520 days, which will provide even more conclusive evidence to support our notion that our crew will be fine for the journey.\cite{15}

Mars is sometimes called the “Red Planet” because of the iron oxide dust that coats the entire surface of the planet. The entire spacecraft, module, and base will be equipped with tight airlocks to ensure that the dust does not become trapped inside of any of the areas where the crew will regularly inhabit.

The variation between very cold and very hot temperatures on the Martian surface will be countered by a stable heating and cooling system within the base.

7.2 Gravitational Accustomation

The change in gravity is seen as the most important health risk of living on Mars, a planet with a fraction of the Earth’s mass. The astronauts will be experiencing a gravity simulation due to centripetal acceleration. When a person stands on Earth, the force of gravity pulls him/her closer to the Earth’s center of mass. What keeps a person standing up is the normal force, or the force of the ground acting on the person to counter gravity. However, on Mars, the forces are are not as strong, and the human body will react differently and uncomfortably. To prevent this from happening all the time on the base, we will use a disk-shaped rotating module for centripetal acceleration. The key to gravity simulation
is that the crew can not see the spinning with the naked eye. It would shatter the illusion. A spinning disk would apply a centripetal force on the person standing on the wall, making the center seem like “up.” When the centripetal acceleration is equal to the acceleration on Earth, or $9.8 \text{ m/s}^2$, the astronauts won’t feel any different than they do on Earth.

### 7.3 Hohmann Transfer vs. Fast Transfer

![Diagram of a fast transfer ellipse between the orbit of Earth and Mars.](image)

**Figure 9:** Diagram of a fast transfer ellipse between the orbit of Earth and Mars.

Having mentioned earlier that the Hohmann Transfer is more efficient than the Fast Transfer, we will see why. Combining the standard elliptic trajectory equations with Lambert’s Theorem allows us to plot a graph of the comparative velocity budget versus the time of flight.

In the calculations section, we show that the velocity budget for the Hohmann ellipse would be $3607 \text{ m/s} + 1865 \text{ m/s} = 5472 \text{ m/s}$ and that the time of flight would be roughly 8.5 months. However, for a fast transfer, the second velocity change at the intersection of the outer orbit and the transfer ellipse does not involve two collinear velocities. Instead, they meet at an angle. Figure 10, generated using an Excel spreadsheet, plots the relative amount of velocity required on the $x$-axis and the relative amount of time required for the flight on the $y$-axis. For example, a trip that requires $6000 \text{ m/s}$ would be plotted with an $x$-coordinate of $\frac{6000-5472}{5472} = 0.0965 = 9.65\%$. The $y$-axis is plotted in a similar manner.

By analyzing the graph, we can see that after the initial increase in velocity budget and thus energy, the corresponding decrease in time of flight becomes very minimal and is not efficient. A 20% decrease in flight time would require a 20% increase in velocity budget. In the case of a 40% decrease in flight time, 90% extra velocity budget would be required.

Detailed calculations and reasoning for the fast transfer graph can be found in *Appendix C.*
Figure 10: Graph comparing time of flight to velocity budget.
8 Related Work

There is a proposed mission to Mars by NASA in February of 2031. Our mission would happen at roughly the same time, but it differs in the types of power that will be used. NASA’s proposed mission will use nuclear energy to power the entire trip, while our mission will send a separate spacecraft two years earlier to collect enough solar energy via solar panels to await the arrival of the astronauts. Nuclear power may also be used, but only to a small degree due to the risk of dangerous explosions or radioactivity. Like our mission, NASA’s mission specifies that the spacecraft will be assembled in space to avoid excessive energy consumption.

9 Conclusion

In the final analysis, we believe that our mission to Mars can be feasible in the near future as technology advances. Our mission can be successful if the right measures are taken to ensure the crew’s safety, the fuel efficiency, and the proper design and method.

With the Hohmann Transfer, solar power, and manufacture of fuel on Mars, the entire mission will be very energy-efficient and reasonable. A simulation conducted last year proved that the crew can survive for 180+ days on Mars, and current experiments project the likely possibility of survival for over 500 days.[14] With further expansion and detail on our designs for the base and spacecraft, as well as the proper funding, our mission will become a reality.

10 Acknowledgements

We would like to thank the 2009 Governor’s School of Engineering and Technology (GSET) program for providing us with the opportunity to explore exciting new topics in engineering, as well as to complete this project. In particular, we would like to thank Justin Meiswinkle, our project mentor, and Kenny Wasserman, our project advisor. We would also like to recognize the NJ Governor’s School of Engineering and Technology (Donald M. Brown, Director, and Blase Ur, Program Coordinator), the Rutgers University School of Engineering (Dr. Yogesh Jaluria, Outgoing Interim Dean, and Dr. Thomas Farris, Dean), and the NJ Governor’s School Board of Overseers for organizing this program. Finally, this program would not have been possible without the generous contributions from our sponsors: Rutgers University, the Rutgers University School of Engineering, the Motorola Foundation, Morgan Stanley, PSEG, Silver Line Building Products, and the families of the 2001-2008 program alumni.
Appendix A: Constants and Equations

In order to more easily describe the equations associated with our journey, this appendix lists some of the more frequently used equations as well as important constants. Furthermore, in order to simplify equations, we will introduce the gravitational parameter, which is equal to the product of the universal gravitational constant and the mass of a specific body.

<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius</td>
<td>Average radius of the specific roughly spherical body</td>
</tr>
<tr>
<td>µ</td>
<td>Gravitational parameter - product of universal gravitational constant and mass of the specific body</td>
</tr>
<tr>
<td>(v_{\text{esc}})</td>
<td>Escape velocity - minimum velocity required at a certain altitude to escape the specific body</td>
</tr>
<tr>
<td>Semimajor axis</td>
<td>Half of longer dimension of ellipse</td>
</tr>
<tr>
<td>(v_{\text{orbit}})</td>
<td>Velocity of smaller body’s orbit around larger body</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Name</th>
<th>Radius ((\times 10^6 \text{ m}))</th>
<th>(\mu) ((\text{m}^3/\text{s}^2))</th>
<th>(v_{\text{esc}}) ((\text{m/s}))</th>
<th>Semimajor axis ((\text{AU})^*)</th>
<th>(v_{\text{orbit}}) ((\text{m/s}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun</td>
<td>696.0</td>
<td>(1.327 \times 10^{20})</td>
<td>617500</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Earth</td>
<td>6.378</td>
<td>(3.986 \times 10^{14})</td>
<td>11180</td>
<td>1.000</td>
<td>29790</td>
</tr>
<tr>
<td>Mars</td>
<td>3.393</td>
<td>(4.297 \times 10^{13})</td>
<td>5032</td>
<td>1.524</td>
<td>24140</td>
</tr>
</tbody>
</table>

\(^*1 \text{AU} = 1.496 \times 10^{11}\) m.

**Forces and Energy**

The magnitude of the attractive force between two masses \(M\) and \(m\) is given by

\[ F = \frac{GMm}{r^2} = \frac{\mu Mm}{r^2}. \]

The magnitude of the centripetal force of a smaller mass \(m\) in circular motion about a larger mass \(M\) is given by

\[ F_c = \frac{mv^2}{r}. \]

The kinetic energy of an object \(m\) moving at velocity \(v\) is given by

\[ K = \frac{1}{2}mv^2. \]
The gravitational potential energy of an object $m$ with respect to a larger body $M$ is given by

\[ U = -\frac{GMm}{r} = -\mu Mm. \]

The total energy of an object $m$ is given by $E = K + U$ and does not change in an isolated system.

**Conic Properties**

![Diagram of ellipse and its properties.](image)

**Figure 11:** Diagram of ellipse and its properties.

For ellipses, the eccentricity is given by $\epsilon = \frac{c}{a}$, where $c$ is the distance from a focus to the center and $a$ is half the length of the longer dimension of the ellipse.

The length of the semi-latus rectum is given by $p = a(1 - \epsilon^2) = \frac{H^2}{\mu}$, where $H$ is the magnitude of the angular momentum.

The total specific energy is given by

\[ E = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}. \]

The distance from one focus to the farther endpoint is given by $r_a$ and the distance from
one focus to the closer endpoint is given by $r_p$. The following formulas apply:

$$r_a = a (1 + \epsilon)$$

$$r_p = a (1 - \epsilon)$$

$$H = r_a v_a = r_p v_p$$

While the above formula for $H$ holds true for the apoapsis and the periapsis of an elliptical orbit, a formula that holds true for all points on the orbit is given by $H = r V \cos \phi$, where $\phi$ is the angle between the tangent velocity vector and the local horizon.

Furthermore, the following formula relates the radius to the semi-latus rectum, eccentricity, and the angle created:

$$r = \frac{p}{1 + \epsilon \cos \nu}.$$  

Rearranging for $\nu$, we find

$$\nu = \cos^{-1} \left( \frac{1}{\epsilon} \left( \frac{p}{r} - 1 \right) \right).$$

An alternative formula for eccentricity is given by

$$\epsilon = \sqrt{1 + \frac{2EH^2}{\mu_S^2}}.$$

**Law of Cosines**

Given three velocity vectors that make up a triangle, their magnitudes $V_1$, $V_2$, $V_3$, and the angle $\alpha$ between vectors $V_1$ and $V_2$, the following formula applies:

$$V_3^2 = V_1^2 + V_2^2 - 2V_1 V_2 \cos \alpha.$$
Appendix B: Mission Calculations

Figure 12: Diagram of Earth and Mars orbiting the Sun. Orbits are approximated as circles to simplify calculations.

First, we will send the satellite into a roughly circular orbit about the Earth at a low-Earth orbit altitude of \( h = 3.571 \times 10^5 \) m, in order to reduce mechanical and fuel requirements. This height is significant because it represents the altitude of the International Space Station, where we will be docking and assembling our spacecraft to take advantage of the low gravitational obstacles. After finishing the assembly at this altitude, the rocket must have enough initial thrusting speed in order to enter the orbit. The energy of this orbit is given by

\[
-\frac{\mu}{2r} = -\frac{\mu}{2(r_E + h)}.
\]

Setting this expression equal to

\[
\frac{v_L^2}{2} - \frac{\mu}{r_E}
\]

yields

\[
v_L = 8112 \text{ m/s}.
\]

However, we can take advantage of Earth’s rotation to reduce engine dependence. The Earth rotates at a velocity of 463.7 m/s at the equator, and the Kennedy Space Center is located at approximately 28.6° north latitude. Furthermore, we will launch at a 23.44° inclination to keep the spacecraft orbiting on the same plane to Mars. The velocity contributed by Earth’s rotation will be

\[
463.7 \text{ m/s } \cos 28.6° \cos 23.44° \text{ m/s } = 374 \text{ m/s},
\]

so that if we launch along the direction of the rotation, we can launch with a velocity of only

\[
8112 \text{ m/s } - 374 \text{ m/s } = 7738 \text{ m/s},
\]
making for a 4.6% savings.

This launch velocity will provide a sufficient amount of centripetal force to keep the satellite at its orbit, so that

\[
\frac{mv_T^2}{r} = \frac{\mu_E m}{r^2} \implies v_T = \sqrt{\frac{\mu_E}{r_E + h}},
\]

so \(v_T = 7693 \text{ m/s}\).

To calculate the escape velocity, we can set the total mechanical energy equal to zero, or

\[
\frac{1}{2}mv_{esc}^2 - \frac{\mu_E m}{r} = 0 \implies v_{esc} = \sqrt{\frac{2\mu_E}{r_E + h}}.
\]

We can note that this is simply \(\sqrt{2}\) times the magnitude of the tangential orbital velocity, so we have \(v_{esc} = 10880 \text{ m/s}\).

After assuming a circular orbit about the Earth, we use the Hohmann transfer to travel to Mars. With the Sun at one focus of the new ellipse, we can find the semi-major axis:

\[
2a = a_E + a_M \implies a = \frac{a_E + a_M}{2},
\]

so \(a = 1.888 \times 10^{11}\). We can ignore the distance between the surface of the Earth and the spacecraft since it is negligibly small compared to the astronomical orbiting distances.

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Figure 13: Diagram of Hohmann transfer used to send spacecraft from orbit of Earth to orbit of Mars.
From here, in order to enter the elliptical Hohmann transfer orbit around the Sun, we need to first escape from the gravitational orbiting pull of the Earth. Now, there are two types of open orbits, or orbits that we can use to escape a planet’s pull. One is a parabolic escape trajectory. This trajectory involves engine thrusting to start the trajectory and then a second thrust at infinity in order to enter the elliptical Hohmann transfer orbit. However, the second option is more energy efficient. With the Oberth maneuver, we can use an Oberth hyperbola and thrust just once to leave a positive residual velocity at infinity to automatically enter the Hohmann transfer ellipse.

Furthermore, using familiar elliptical equations, we can calculate the velocities of the elliptical orbit at the perihelion and the aphelion. From Figures 11 and 13, we know that

\[ c = a - a_E = 3.92 \times 10^{10} \text{ m}. \]

From this we find that

\[ \epsilon = \frac{c}{a} = 0.208. \]

Then the length of the semi-latus rectum is given by

\[ p = a (1 - \epsilon^2) = 1.806 \times 10^{11} \text{ m}. \]

Finally,

\[ v_p = \frac{H}{r_p} = \frac{\sqrt{p\mu S}}{r_p} = 32724 \text{ m/s} \]

and

\[ v_a = \frac{H}{r_a} = \frac{\sqrt{p\mu S}}{r_a} = 21472 \text{ m/s}. \]
In order to enter this elliptical orbit, we would fire two thrusts, once at the perihelion to push the spacecraft into a Hohmann transfer orbit and a second time at the aphelion to circularize the orbit to come close to Mars. Our desired residual velocity is equal to the difference between the velocity of the spacecraft around Earth and Earth’s orbital velocity, or

\[ v_p - v_E = 32724 \text{ m/s} - 29800 \text{ m/s} = 2924 \text{ m/s}. \]

We can calculate the required thrusting velocity using the law of conservation of energy:

\[
\frac{1}{2} m v^2_{\infty} - \frac{\mu}{r_{\infty}} = \frac{1}{2} m v^2_r - \frac{\mu}{r}.
\]

Since \( \frac{\mu}{r_{\infty}} \) tends to zero as \( r_{\infty} \) tends to \( \infty \), we can eliminate that term and rearrange to get

\[ v_r = \sqrt{\frac{2\mu}{r} + v^2_{\infty}}. \]

Since \( \frac{2\mu}{r} = v^2_{\text{esc}} \) at \( r \), we have

\[ v_r = \sqrt{v^2_{\text{esc},r} + v^2_{\infty}} = 11266 \text{ m/s}. \]

However, since the speed of the circular orbit about the Earth is already 7784 m/s, it is necessary to boost the spacecraft by only

\[ 11266 \text{ m/s} - 7784 \text{ m/s} = 3482 \text{ m/s}. \]

To determine the flight time of this Hohmann transfer ellipse, we use Kepler’s third law of planetary motion, which states that the square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit. In other words,

\[ \frac{T^2}{a^3} = k. \]

Using the Earth as a model, we have that the period of the Earth is 1 year and the semi-major axis is 1 AU, so

\[ T^2 = a^3 \implies T = a^{3/2}. \]

We convert our calculated value for the semi-major axis to AU to get

\[ \frac{1.888 \times 10^{11} \text{ m}}{1.49598 \times 10^{11} \text{ m/AU}} = 1.262 \text{ AU}. \]

Plugging this into the formula,

\[ T = 1.4177 \text{ years}. \]

The time to complete the Hohmann transfer ellipse is half this, or

\[ T_{1/2} = 0.70886 \text{ years} = 8.5 \text{ months}. \]
Figure 15: Diagram of Oberth hyperbolic flyby used to enter an areostationary orbit about Mars.

After the 8.5 months traveling time, the spacecraft will arrive at Mars. In order to land on Mars, we reverse the launching process used to escape Earth’s surface. Since the spacecraft is moving at 21472 m/s and Mars is revolving at a velocity of 24077 m/s, the spacecraft is moving at a velocity $24077 \text{ m/s} - 21472 \text{ m/s} = 2605 \text{ m/s} = v_\infty$ slower with respect to Mars. We use a hyperbolic flyby in order to enter a circular orbit about Mars.

We want the satellite to remain over one spot on Mars so that there will be easy communication between the satellite and the base on the surface. To calculate the radius of the satellite about the center of Mars, we use Kepler’s third law of planetary motion:

$$T = 2\pi \sqrt{\frac{r^3}{\mu_M}} \implies r = \sqrt[3]{\frac{\mu_M T^2}{4\pi^2}},$$

which evaluates to

$$2.0438 \times 10^7 \text{ m}.$$

Using conservation of energy, we have

$$\frac{1}{2}mv_\infty^2 - \frac{\mu_M}{r_\infty} = \frac{1}{2}mv_p^2 - \frac{\mu_M}{r_p},$$

from which

$$v_p = \sqrt{v_\infty^2 + \frac{2\mu_M}{r}} = 3315 \text{ m/s}.$$ 

When the spacecraft nears Mars, the gravitational force provides a sufficient centripetal force so that

$$\frac{mv^2}{r} = \frac{\mu_M m}{r^2} \implies v = \sqrt{\frac{\mu_M}{r}}.$$
which evaluates to 1450 m/s.

Thus, the spacecraft must decrease its speed by

\[ 3315 \text{ m/s} - 1450 \text{ m/s} = 1865 \text{ m/s}. \]

Finally, the total velocity budget for the trip is

\[ 8112 \text{ m/s} + 3482 \text{ m/s} + 1865 \text{ m/s} = 13459 \text{ m/s}. \]

Then, by using the same technique to calculate the launching velocity, we can calculate the velocity upon impact of the spacecraft on Mars if no parachutes or thrusts are used. Using the conservation of energy, we have

\[
\frac{1}{2}mv^2 - \frac{m\mu_M}{r} = \frac{1}{2}mv_{impact}^2 - \frac{m\mu_M}{r_M},
\]

from which we obtain

\[ v_{impact} = 4850 \text{ m/s}. \]

To dramatically decrease this velocity, we will fire engines in the reverse direction and then use a grape-like landing apparatus similar to the one used for the Spirit and Opportunity mission to land at a low terminal velocity.
Appendix C: Fast Transfer Analysis Calculations

Figure 16: Diagram of a fast transfer ellipse between the orbit of Earth and Mars.

To evaluate the energy efficiency of increasing the major axis of the transfer ellipse and thus lowering the time of flight, we plotted a graph of the efficiency.

The method starts with defining an arbitrary semi-major axis greater than or equal to the Hohmann transfer ellipse’s semi-major axis. Given an arbitrary $a$, we can find the specific energy, or

$$E_t = -\frac{\mu}{2a_t}. $$

Then, by rearranging the equation for total mechanical energy, we can find that

$$V_{ti} = \sqrt{2\left(E + \frac{\mu S}{r_i}\right)},$$

where $r_i$ is the radius of the orbit of Earth around the Sun, or 1 AU. With this information, we can evaluate $\Delta V_1$. The earth’s orbital velocity is 29790 m/s. Since this velocity is collinear with that of the periapsis of the transfer ellipse, we can subtract them to find that

$$\Delta V_1 = V_{ti} - 29790 \text{ m/s}.$$

The eccentricity of the transfer ellipse is given by

$$\epsilon = \sqrt{1 + \frac{2EH^2}{\mu_S^2}}.$$

Now, we take note that the angular momentum is conserved throughout this trip. The angular momentum at the periapsis is

$$H = rV \cos \phi = r_i V_{ti} \cos 0 = r_i V_{ti}.$$
Using this value, we can find the velocity at the intersection of the transfer orbit and the orbit of Mars to be

$$V_{to} = \sqrt{2 \left( E + \frac{\mu}{r_o} \right)}.$$  

The elevation angle at this intersection is then represented by

$$\cos \phi = \frac{H}{r_o V_{to}}.$$  

Unlike calculating $\Delta V_1$, calculating $\Delta V_2$ is not as easy because the two velocity vectors are not collinear. Using the law of cosines, we find that

$$\Delta V_2 = \sqrt{V_{to}^2 + V_o^2 - 2V_{to}V_o \cos \alpha},$$

where $V_o$ is the orbital velocity of Mars.

Then the velocity budget is $\Delta V_T = \Delta V_1 + \Delta V_2$. Thus, we have calculated, through a series of equations, the velocity budget for a given semi-major axis $a$. Now, it is necessary to calculate the associated time of flight. We start with finding the angle covered during the transfer ellipse flight. First, we find the semi-latus rectum, or

$$p = \frac{H^2}{\mu S}.$$  

The angle is then given by

$$\nu = \cos^{-1} \left( \frac{1}{\epsilon} \left( \frac{p}{r_o} - 1 \right) \right).$$

Lambert’s Theorem states that the time of flight between two points on an elliptical orbit is given by

$$TOF = \sqrt{\frac{a^3}{\mu S} \left( (\alpha - \sin \alpha) - (\beta - \sin \beta) \right)},$$

where

$$\cos \alpha = 1 - \frac{s}{a}$$

and

$$\cos \beta = 1 - \frac{s - c}{a},$$

where $s$ is the semi-perimeter of the triangle created by the two radius vectors and the chord spanning them, or

$$s = \frac{r_i + r_o + d}{2}.$$  

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Using the law of cosines, \( d \) is given by

\[
d = \sqrt{r_i^2 + r_o^2 - 2r_i r_o \cos \theta},
\]

where \( \theta \) is the angle between the two radius vectors.

With all our known constants and derived equations, we can plot some points to graph velocity budget as a function of time of flight. This graph can be found as Figure 10 under \textit{Engineering Analysis}. 
References


